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STUDY OF A QUEUEING SYSTEM WITH A FINITE RANGE SERVICE TIME DISTRIBUTION

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Abstract

In the present paper an attempt has been made to analyse a single server waiting line system with finite range model, which is a well known life testing model.

INTRODUCTION

In the paper we estimate the parameters involved in a single server waiting line system with the service time distribution as a finite range model namely, Mukheerji-Islam model, which is a well known life testing model.

Consider a single server queuing with infinite capacity having FCFS (First Come First Serve) queue discipline. We assume that the arrivals are Poisson with arrival rate λ. But the service time distribution of the process is a new finite range probability distribution which is introduced by Mukherjee-Islam (1983) as a life testing model.

$$f(t;\ \theta,p) = (p/\theta^p)\ t^{p-1}; \qquad p,\ \theta > 0; \quad t \geq 0 \qquad \text{....(1)}$$

The above model is monotonic decreasing and highly skewed to the right. The graph is J-shaped thereby showing the unimodel feature. The distribution function of above model will be

$$F(t) = [t/\theta]^p \qquad \text{....(2)}$$

with $\quad \text{Mean} = \dfrac{p}{p+1}.\theta$

and Variance = $\frac{p}{(p+1)^2(p+2)}.\theta^2$

MAXIMUM LIKELIHOOD ESTIMATES

Consider a random sample $T_1, T_2, \ldots, T_n$ from the population with p.d.f. (1). The likelihood function is given as

$$L(t;\theta,p) = p^n \theta^{-np} \prod_{i=1}^{n} t_i^{p-1} \qquad \ldots(3)$$

Taking log on both the sides, we get

$$\log L = n \log p - np \log \theta + (p-1)\Sigma \log t_i \qquad \ldots(4)$$

Differentiating the equation (4) partially with respect to 'p' and equating it to zero,

$$\frac{\partial \log L(t)}{\partial p} = \frac{n}{p} - n\log\theta + \sum \log t_i = 0$$

The m.l.e. of p is finally obtained as

$$\hat{p} = \frac{n}{n\log\theta - \sum \log t_i} \qquad \ldots(5)$$

Again, differentiating the equation partially (4) with respect to 'θ' and equating it to zero to obtain the m.l.e. of θ

$$\frac{\partial \log L(t)}{\partial \theta} = \frac{np}{\theta} = 0$$

In the solution for MLE of θ the traditional method is not applicable. The MLE is obtained through order statistic technique. Since the upper limit of the model is θ, it is convincing to take $t_{(n)}$ i.e. maximum t_i as the m.l.e for the parameter θ

i.e. $\hat{\theta} = t_{(n)} = \max(t_1, t_2, \ldots, t_n)$ ....(6)

ANALYSIS OF THE MODEL

To analyze the model we will obtain probability generating function of H_n, the probability that there are n arrivals during the service time of a customer.

Let H_n be the probability that there are n arrivals during the service time of a customer. Let H(z) denotes the probability generating function (p.g.f.) of H_n given as

$$H(z) = \sum_{n=1}^{\infty} H_n z^n \; ; |z| \leq 1$$

Following heuristic argument of Kendall (1953) and Gross and Hariss (1974), the probability H_n that there are n arrivals during the service time is given by

$$H_n = \int_0^{\theta} \frac{e^{-\lambda t}(\lambda t)^n}{n!}\left(\frac{p}{\theta^p}\right) t^{p-1}\,dt \qquad \text{....(7)}$$

Then the probability generating function of H_n is

$$H(z) = \sum_{n=0}^{\infty} z^n \int_0^{\theta} \frac{e^{-\lambda t}(\lambda t)^n}{n!}\left(\frac{p}{\theta^p}\right) t^{p-1}\,dt$$

$$= \left(\frac{p}{\theta^p}\right)\int_0^{\theta} \sum_{n=0}^{\infty} z^n \frac{e^{-\lambda t}(\lambda t)^n}{n!} t^{p-1}\,dt$$

$$= \left(\frac{p}{\theta^p}\right)\int_0^{\theta} e^{-\lambda t} \sum_{n=0}^{\infty} \frac{(\lambda zt)^n}{n!} t^{p-1}\,dt$$

$$= \left(\frac{p}{\theta^p}\right)\int_0^{\theta} e^{-\lambda t} e^{\lambda z t}\, t^{p-1}\,dt$$

$$= \left(\frac{p}{\theta^p}\right)\int_0^{\theta} e^{-(\lambda-\lambda z)t}\, t^{p-1}\,dt$$

$$= \left(\frac{p}{\theta^p}\right)\int_0^{\theta} \sum_{j=0}^{\infty} \frac{(-(\lambda-\lambda z)t)^j}{j!} t^{p-1}\,dt$$

$$= \left(\frac{p}{\theta^p}\right)\sum_{j=0}^{\infty} \frac{(-(\lambda-\lambda z))^j}{j!} \int_0^{\theta} t^{p+j-1}\,dt$$

$$H(z) = p.\sum_{j=0}^{\infty} \frac{(-(\lambda-\lambda z))^j}{j!}\,\frac{\theta^j}{p+j} \qquad \text{....(8)}$$

The average number of arrivals during the service time is

$$H'(z)\Big|_{z=1} = \frac{p}{p+1}.\theta\lambda \qquad \text{....(9)}$$

Let we denote that $\mu = \frac{p+1}{p}.\theta$ (the reciprocal of the mean) then

$$H'(z)\Big|_{z=1} = \frac{\lambda}{\mu} \qquad \text{....(10)}$$

Now, let P_n be the probability that there are 'n' customers in the system at the steady state and P(z) be the probability generating function of P_n. Then by expanding P(z) and collecting the coefficient of z^n, we get P_n.

Furthermore, the analysis can be carried out in the same manner as in the Pathak (1995) for inversegaussian service time distribution system.

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